

# AdS/CFT Correspondence Beyond its Supergravity Approximation

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## Abstract

We will study the AdS/CFT correspondence in an intermediate region between the strong form of this correspondence (string theory on AdS being dual to a boundary CFT), and the weak form of this correspondence (supergravity on AdS being dual to a boundary CFT). We will go beyond the supergravity approximation in the AdS by using the fact that strings have an extended structure. We will also calculate the CFT dual to such string corrections in the bulk, and demonstrate that they are consistent with the strong form of the AdS/CFT correspondence. So, even though the conformal dimensions of both the relevant and the irrelevant operators will receive string corrections, the conformal dimension of marginal operators will not receive any such corrections.

## 1 Introduction

According to the AdS/CFT duality, superstring theory on AdS is dual to a superconformal field theory on its boundary [1]-[4]. This full correspondence is called the strong AdS/CFT correspondence. However, it is very difficult to analyse this duality between the string theory and the boundary conformal field theory. This is because it is very difficult to analyse non-perturbative aspects of the string theory. In fact, even at string tree level all aspects of string theory are not completely understood. So, a weaker form of the AdS/CFT correspondence is usually used. This is done by restricting to the analyses to the low energy effective field theory description of the string theory. It is known that the supergravity theories are low energy effective field theories obtained from the string theory [5]-[9]. So, according to the weak form of the AdS/CFT correspondence, the supergravity on AdS is dual to the boundary conformal field theory [10]-[13]. In this paper, we will analyse an intermediate region between the weak form and the strong form of the AdS/CFT correspondence. We still cannot analyse the full string theory side of the duality, however, in this paper,

we will analyse the leading order corrections to the supergravity approximation coming from the string theory.

We will go beyond the supergravity approximation by using the fact that strings have an extended structure. As strings have an extended structure, the spacetime cannot be probed below the length scale of the fundamental string [14]. This is because the fundamental string is the smallest probe available in string theory, and so it is not possible to probe anything smaller than the fundamental string. Thus, string theory naturally comes equipped with a minimum length scale. It is known that if a minimum measurable length scale exists in any theory, then the coordinate representation of the momentum operator will get deformed in that theory [15]-[22]. Furthermore, this result has been generalized to a covariant quantum field theory [23]-[25]. This deformation occurs due to the generalized uncertainty principle [26]-[31]. The generalized uncertainty principle is usually motivated from quantum gravity, and the Planck length is taken as the minimum measurable length scale. However, in this paper, we will use the generalized uncertainty principle as a tool for analysing the string corrections going beyond the supergravity approximation. It may be noted that we will only be using the formalism developed by generalized uncertainty principle as a tool for analysing the string corrections, and the main aim of this paper is to show that such corrections are consistent with the strong form of the AdS/CFT correspondence. Hence, in this paper, we will be analysing the AdS/CFT correspondence beyond its supergravity approximation.

Now as the string theory comes naturally equipped with a minimum measurable length scale, we will expect that the low energy effective field theory derived from the string theory will also be deformed because of this minimum measurable length scale (string length scale). The correction terms produced because of such a deformation will be proportional to the string length scale, as they are produced because of the extended structure of strings. In this paper, we will analyse such a deformation of a field theory in the bulk. We will simplify our analysis by considering a simple scalar field theory in the bulk. It may be noted that even though the field content of the full supergravity theories will be more complicated than this simple scalar field theory, the basic ideas developed here can be applied to the full supergravity theory. Thus, it will be possible to obtain a similar deformation for all fields in a supergravity theory. As these deformations will be proportional to the length of the fundamental string, they can be considered as corrections terms that will be generated by going beyond the supergravity approximation. The main features of such an analysis can also be understood analysing the simple example used in this paper. It may be noted that the CFT dual to a scalar field theory on the AdS has been studied in the context of algebraic holography [32]-[33].

So, in this paper, we will first deform a scalar field theory by using the fact that strings have an extended structure. This deformation will be proportional to the length of the strings as it will occur because of the extended structure of strings. Furthermore, as these correction terms will be proportional to the length of the string, they can be considered as higher order correction generated from the string theory. After obtaining the corrections terms for the bulk scalar field theory, we will use the AdS/CFT for calculating the boundary dual to such correction terms. We will observe that the conformal dimension of both the relevant and irrelevant operators will change due to such a deformation. However, the conformal dimension of the marginal operators will not receive any

corrections. This is something that we would expect from the AdS/CFT correspondence, according to the AdS/CFT correspondence the full string theory on AdS (not only its supergravity approximation), is dual to a superconformal field theory on the boundary. So, this analysis can be seen as a test of AdS/CFT correspondence going beyond the supergravity approximation.

## 2 Leading Order String Corrections

In this section, we will analyse the leading order string correction for a field theory on AdS. This field theory will be viewed as a part of some low energy effective field theory obtained from string theory. The Polyakov action of string theory can be written as

$$S = \frac{1}{4\pi\alpha'} \int d^2z \mathcal{L}, \quad (1)$$

where  $\mathcal{L}$  is the Polyakov Lagrangian for the string theory, and the string tension  $4\pi\alpha'$  is equal to square of the string length. The important feature of this action is that as it is the action of an extended body, it is not possible to use the strings described by this action to probe the spacetime at distances smaller than this string length scale [14]. It may be noted that it is possible to obtain the conventional quantum mechanics as a low energy effective field theory approximation to string theory [34]. In this analysis the extended structure of strings is neglected. It may be noted that the extended structure of strings is not consistent with the usual Heisenberg uncertainty principle because according to the usual Heisenberg uncertainty principle, the length can be measured with arbitrary precision, if the momentum is not measured. So, it is not possible to probe spacetime below the string length scale because of the extended structure of strings. However, it is possible to generalize the usual Heisenberg uncertainty principle to a generalized uncertainty principle which restricts the measurement of the length to a minimum value [26]-[31]. This generalized uncertainty principle deforms the usual Heisenberg algebra, and the deformation of the Heisenberg algebra in turn deforms the coordinate representation of the momentum operator [17]-[21]. This deformed momentum operator has been used for studying quantum field theory on spacetime with a minimum measurable length scale [23]-[25]. It may be noted that such an analysis has been motivated from quantum gravity, and hence the Planck length is taken as the minimum measurable length scale. However, in this paper, we will use the formalism developed in generalized uncertainty principle, for analysing the low energy effective field theory approximation to the string theory, and we will take the string length scale as the minimum measurable length scale. Even though the string length scale is taken to be identical with the Planck length scale, there is no reason to assume that, and it is consistent to take a large length scale as the string length scale. It may be noted that we will only be using this formalism as a tool for analysing the string theory effects going beyond the supergravity approximation. So, the main motivation of this paper is to test the AdS/CFT correspondence beyond its supergravity limit.

Now we can write the deformation of the low energy Heisenberg algebra for a particle. Now let us start with a particle whose Heisenberg algebra,  $[X, P] = i$ , is deformed because of the extended structure of strings. As this deformation occurs due to the extended structure of strings, so it will be proportional to

some function of the string length which measures the extended structure of the strings. Thus, we can write  $[X, P] = i[1 + f(4\pi\alpha')(3P^2)]$ , where  $f(4\pi\alpha')$  is a suitable function of the fundamental length scale, such that  $f(4\pi\alpha') \rightarrow 0$  in the limit  $4\pi\alpha' \rightarrow 0$ . This is because this deformation occurs due to the extended structure of strings, and so it will vanish when we neglect such an extended structure. We will assume the simplest form of such a function  $f(4\pi\alpha') = 4\pi\alpha'$ , and write  $[X, P] = i[1 + 4\pi\alpha'(3P^2)]$ . This deformation of the Heisenberg algebra causes a deformation of the usual Heisenberg uncertainty relation  $\Delta P \Delta X \geq [1 + 4\pi\alpha'\Delta P^2]/2$ . Thus, we can write  $\Delta P \leq (4\pi\alpha')^{-1}[\Delta X \pm \sqrt{\Delta X^2 - 4\pi\alpha'}]$ , and this implies the existence of a minimum measurable length  $l_{min}$  for the particle,

$$\Delta X \geq l_{min} = \sqrt{4\pi\alpha'}. \quad (2)$$

Thus, we cannot probe the spacetime below the string length scale, and so this new deformed Heisenberg algebra is consistent with the extended structure of the strings. It may be noted that it is possible to perform a similar deformation of the usual Heisenberg algebra in higher dimensions. So, motivated by the quantum field theory on a background with a minimum measurable length scale [23]-[25], the deformation of the Heisenberg algebra  $[X^M, P_N] = i\delta_N^M$ , by the string length scale can be written as

$$[X^M, P_N] = i\delta_N^M + 4\pi\alpha' [\delta_N^M g^{PQ} P_P P_Q + 2iP^M P_N]. \quad (3)$$

The uncertainty relation consistent with this deformed Heisenberg algebra will again predict the existence of a minimum length which is related to the string length,  $l_{min} = \sqrt{4\pi\alpha'}$  [26]-[25]. So, this deformation of the Heisenberg algebra does restrict the measurement of the length scale to string theory length scale. Hence, this deformation is consistent with the existence of an extended structure for strings. This deformation of the Heisenberg algebra also causes a deformation of the momentum operators to

$$P_M^{(\alpha')} = P_M (1 + 4\pi\alpha' g^{NP} P_N P_P), \quad (4)$$

where,  $P_N = -i\nabla_N$ , is the original un-deformed momentum. Now we define the operator  $P_M^{(\alpha')} = -i\nabla_M^{(\alpha')}$  to be the coordinate representation of the deformed momentum operator. Hence, we can write

$$\nabla_M^{(\alpha')} = \nabla_M (1 - 4\pi\alpha' g^{PQ} \nabla_P \nabla_Q). \quad (5)$$

Now let us consider a massive free scalar field action

$$S = -\frac{1}{2} \int d^{d+1}x \sqrt{g} [g^{MN} \nabla_M \phi \nabla_N \phi + m^2 \phi^2]. \quad (6)$$

We will assume this to be a part of some low effective field theory action obtained from string theory. Thus, we will now analyse the leading order corrections to this action that will occur due to the extended structure of strings. In order to do that, we will first express this action as

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{g} \phi [g^{MN} \nabla_M \nabla_N - m^2] \phi. \quad (7)$$

As this action is thought to be a part of some low energy effective field theory derived from the Polyakov action of string theory, it gets deformed due to the

existence of the extended structure of strings in the Polyakov action of string theory. This deformed action can now be written as

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{g} \phi [g^{MN} \nabla_M^{(\alpha')} \nabla_N^{(\alpha')} - m^2] \phi. \quad (8)$$

Comparing the deformed and the un-deformed momentum, we can also write the equation for the deformed free scalar field theory as

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{g} \phi [\mathcal{D} - m^2 - 8\pi\alpha' \mathcal{D}^2] \phi, \quad (9)$$

where  $\mathcal{D} = g^{PQ} \nabla_P \nabla_Q$ . It may be noted that this is exactly the form of the action that would have been obtained from an derivative expansion of the massive free scalar field theory in the framework of effective field theory [35]-[40]. This is because according to the effective field theory, the low energy effective field theory can be written as

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{g} \phi [\mathcal{D} - m^2 - \frac{1}{\Lambda} \mathcal{D}^2] \phi, \quad (10)$$

where  $\Lambda$  is the scale which is integrated out. As we cannot gain any information beyond the string length scale, we have to integrate it out in an effective field theory. Hence, we can identify  $\Lambda^{-1} = 8\pi\alpha'$ , and arrive at the same deformation using the derivative expansion in the effective field theory.

### 3 CFT Dual

In the previous section, we obtained higher derivative corrections to the action of a scalar field theory due to the extended structure of strings. This was to the first order in  $4\pi\alpha'$ . In this section, we will analyse the CFT dual to such corrections terms in the bulk. The equation of motion for this scalar field is given by

$$(\mathcal{D} - m^2)\phi - 8\pi\alpha' \mathcal{D}^2 \phi = 0, \quad (11)$$

The metric for the AdS in the Poincaré patch can be written as

$$ds^2 = \frac{L^2}{z^2} (dz^2 + \delta_{\mu\nu} dx^\mu dx^\nu), \quad (12)$$

where  $L$  is the radius of the AdS space. So, the Laplacian on AdS can be written as

$$\begin{aligned} \mathcal{D} &= g^{PQ} \nabla_P \nabla_Q \\ &= L^{-2} z^2 (\partial_z^2 - (d-1) z^{-1} \partial_z + \delta_{\mu\nu} \partial_\mu \partial_\nu). \end{aligned} \quad (13)$$

It may be noted that the term  $8\pi\alpha' \mathcal{D}^2 \phi$  is generated by string theory effects going beyond its supergravity approximation. This term deforms the equation of motion of the scalar field in AdS into a fourth order differential equation. However, the extra boundary conditions needed for it are obtained by making the boundary terms appearing in the variation of the action vanish.

Now we will obtain the solution of Eq. (11). To do that we will use the Fourier transform of  $\phi(z, x)$  in  $x^\mu$  coordinates

$$\phi(x, z) = \int \frac{d^d k}{(2\pi)^d} e^{ik \cdot x} f_k(z). \quad (14)$$

We will also use the ansatz  $f_k(z) \approx z^\Delta$ , for solutions close to the boundary i.e., we will keep only the leading terms near  $z = 0$ . It may be noted that the conformal dimension for the boundary CFT is denoted by  $\Delta$ , and the boundary is defined at  $z = 0$ . Thus, we obtain the following equation

$$\Delta(\Delta - d) - m^2 L^2 + \frac{8\pi\alpha'}{L^2} (-\Delta^4 + 2d\Delta^3 - d^2\Delta^2) = 0. \quad (15)$$

It may be noted that we have only kept the leading order terms near  $z = 0$  to obtain this result. As the complex roots give oscillatory solutions near the boundary and this in turn makes the action un-bounded, so we will only consider real roots for this equation. This motivates us to define  $\epsilon(\alpha') = 8\pi\alpha'/L^2$  as the dimensionless parameter governing the string correction beyond its supergravity approximation. Notice that  $\epsilon(\alpha') \ll 1$  in the regime where we can still trust the supergravity solution in the bulk. In order for Eq. (15) to have only real roots, its discriminant  $\Gamma$  must be bigger than zero. The discriminant up to order  $\epsilon(\alpha')$ , is given by

$$\Gamma = (4d^2 + 16L^2m^2)\epsilon(\alpha') + \mathcal{O}(\epsilon(\alpha')^2). \quad (16)$$

The condition  $\Gamma > 0$  gives the usual Breitenlohner-Freedman (BF) bound  $m^2 > -d^2/4L^2$ . To order  $\epsilon(\alpha')^2$ , one obtains the following result,

$$\begin{aligned} \Gamma = & -128L^4\epsilon(\alpha')^2m^4 + (16L^2\epsilon(\alpha') - 32d^2L^2\epsilon(\alpha')^2)m^2 \\ & + 4d^2\epsilon(\alpha') + d^4\epsilon(\alpha')^2. \end{aligned} \quad (17)$$

The condition  $\Gamma > 0$  can now be expressed as

$$f_1(\epsilon(\alpha'))m^4 + f_2(\epsilon(\alpha'))m^2 + f_3(\epsilon(\alpha')) < 0, \quad (18)$$

where

$$f_1(\epsilon(\alpha')) = 128\epsilon(\alpha')L^4, \quad (19)$$

$$f_2(\epsilon(\alpha')) = -16L^2\epsilon(\alpha') + 32d^2L^2\epsilon(\alpha')^2 \quad (20)$$

$$f_3(\epsilon(\alpha')) = -4d^2\epsilon(\alpha') - d^4\epsilon(\alpha')^2. \quad (21)$$

Thus, the condition  $\Gamma > 0$  can be written as

$$\frac{-f_2 - \sqrt{f_2^2 - 4f_1f_3}}{2f_1} < m^2 < \frac{-f_2 + \sqrt{f_2^2 - 4f_1f_3}}{2f_1}. \quad (22)$$

This is the modified BF bound due to the string theory effects in AdS beyond the supergravity approximation. Now, to the order  $\epsilon(\alpha')$ , we obtain the following result

$$-\frac{d^2(4 + d^2\epsilon(\alpha'))}{16L^2} < m^2 < \frac{d^4\epsilon(\alpha')}{16L^2} + \frac{1}{8L^2\epsilon(\alpha')}. \quad (23)$$

The correction to the lower bound implies that more tachyonic modes are allowed in AdS especially in the deep stringy regime  $\epsilon(\alpha') \approx \mathcal{O}(1)$ . The roots of Eq. (15) can be explicitly written as

$$\Delta_1 = \frac{1}{2}(d - \sqrt{d^2 + m^2 L^2}) + \frac{L^4 m^4 \epsilon(\alpha')}{\sqrt{d^2 + 4L^2 m^2}} + \mathcal{O}(\epsilon^2), \quad (24)$$

$$\Delta_2 = \frac{1}{2}(d + \sqrt{d^2 + m^2 L^2}) + \frac{L^4 m^4 \epsilon(\alpha')}{\sqrt{d^2 + 4L^2 m^2}} + \mathcal{O}(\epsilon^2), \quad (25)$$

$$\Delta_3 = \frac{1}{\sqrt{\epsilon(\alpha')}} + \frac{d}{2} + \frac{1}{8}(d^2 - 4L^2 m^2) \epsilon(\alpha')^{1/2} + \mathcal{O}(\epsilon^{3/2}), \quad (26)$$

$$\Delta_4 = -\frac{1}{\sqrt{\epsilon(\alpha')}} + \frac{d}{2} - \frac{1}{8}(d^2 - 4L^2 m^2) \epsilon(\alpha')^{1/2} + \mathcal{O}(\epsilon^{3/2}). \quad (27)$$

Near the boundary  $z = 0$ , a classical solution will behave as

$$\phi(z, x) = z^{\Delta_1} A_1(x) + z^{\Delta_2} A_2(x) + z^{\Delta_3} A_3(x) + z^{\Delta_4} A_4(x). \quad (28)$$

For the range of masses within the modified BF bound given by Eq. (23), we have  $\Delta_1 < \Delta_2$ . Furthermore, both the roots  $\Delta_3$  and  $\Delta_4$  are non-analytic in  $\epsilon(\alpha')$ . If we consider the boundary condition,  $A_1(x) = A_3(x) = A_4(x) = 0$ , then the boundary behavior of  $\phi(z, x)$  is determined by  $\Delta_2$  and in this case the field  $\phi(z, x)$  maps to a boundary operator with conformal dimensions  $\Delta_2$ . It may be noted that from the fact that the full string theory is dual to the superconformal field theory on the boundary, the conformal dimension of the marginal operator dual to  $\phi(z, x)$  is expected not to receive any string corrections up to any order in  $\epsilon(\alpha')$ . In fact, one can easily check from the explicit formula of  $\Delta_2$ , that  $\Delta_2(m^2 = 0) = d$  to all orders in  $\epsilon(\alpha')$ , i.e., the conformal dimension does not receive any string correction from effects beyond its supergravity approximation. This was expected to be the case since  $\mathcal{N} = 4$  SYM theory is conjectured to be dual to the full string theory in the bulk according to the strong AdS/CFT correspondence, and the CFT operator dual to a massless scalar  $\phi(z, x)$  in the bulk is  $Tr(F^2)$ , which is an exactly marginal operator in  $\mathcal{N} = 4$  SYM. In this paper, we have explicitly demonstrated that this result holds beyond the supergravity approximation. However, conformal dimensions of the irrelevant operators ( $m^2 > 0$ ) and the relevant operators ( $m^2 < 0$ ), on the boundary CFT, do receive string correction of order  $\epsilon(\alpha')$ . We can repeat this analysis for other fields in AdS and use it for studying AdS/CFT correspondence. We again expect that the marginal operators will not receive any string corrections. It may be noted that CFT operators dual to the purely stringy states from strong AdS/CFT correspondence are expected to scale  $N^{2/3}$  in five dimensions, or  $N^{1/4}$  in ten dimensions [41]. This is exactly the scaling behavior of the new CFT operators dual to the deformed scale field theory. Hence, we can conclude that this deformation does actually represent phenomena going beyond the supergravity approximation. Finally, according to the strong AdS/CFT correspondence,  $\mathcal{N} = 4$  SYM is dual to the full string theory on AdS, so the supersymmetry of the boundary field theory dual cannot break by these string theory effects in the bulk theory. Thus, if we consider such  $4\pi\alpha'$  corrections to the supergravity approximation of the string theory, we will still obtain an  $\mathcal{N} = 4$  SYM on the boundary. This is because string theory effects going beyond supergravity approximation cannot break the supersymmetry of the

boundary theory [42]-[45]. So, the supersymmetry cannot be broken by the analysing done in this paper. We would like to point out that in this paper, we have only analysed the leading order string corrections, however, even the non-perturbative string corrections cannot break supersymmetry of the boundary theory because of the strong form of the AdS/CFT correspondence.

## 4 Conclusion

In this paper, we analysed the AdS/CFT correspondence beyond its supergravity approximation. We still did not analyse this correspondence for the full string theory on AdS, but we analysed it for the leading order corrections obtained from the string theory. This was done by using that fact that strings have an extended structure. This extended structure of strings prevented them to probe spacetime at arbitrary small length scales. The existence of such an extended structure was used to deform a scalar field theory on AdS. Such a scalar field theory was assumed to be a part of some effective field theory approximation to the string theory. Even though the full supergravity action would have a very complicated field content, this simple example was used to understand the general features of such corrections going beyond the supergravity approximation. Hence, the formalism of generalized uncertainty was used as a tool to obtain string theory corrections to a scalar field theory on AdS. Thus, we were able to explicitly obtain higher derivative corrections to the low energy massive free scalar field theory on AdS.

We also analysed the boundary dual theory to this deformed scalar field theory on the bulk using the AdS/CFT correspondence. As these corrections were generated from string theory, we were able to analyse the CFT dual to the string theory going beyond its supergravity approximation. We were able to demonstrate that the conformal dimensions of both the relevant and the irrelevant operators receive correction from these string theory effects going beyond its supergravity approximation. However, the conformal dimension of marginal operators do not receive any such corrections. This was expected to occur as the full string theory on AdS (not just its supergravity approximation) is dual to the  $\mathcal{N} = 4$  super-Yang-Mills theory according to strong AdS/CFT correspondence. We also obtained the CFT operators are dual to the purely stringy states. This was done by observing that they have the right scaling behavior.

Finally, it will be interesting to extend this work to various supergravity theories. Even though the field context of the supergravity theories would be more complicated, we expect that this procedure can be repeated for the full supergravity theories [5]-[9]. We also expect similar results to hold for the full supergravity action. However, it will be interesting to explicitly construct such actions, and demonstrate this result explicitly. It may be noted that the UV divergences of the correlation functions of the boundary conformal field theory are related to the IR divergences of the supergravity. The IR divergences on the gravitational side are the same as near-boundary effects. So, they can be analysed using the holographic renormalization [46]-[50]. In this approach the cancellation of the UV divergences does not depend on the IR physics. Thus, the holographic renormalization will only depend on the near-boundary analysis. It would be interesting to use the holographic renormalization for analysing the



string corrections going beyond the supergravity approximation.

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